Tracking Functional Data by Nonparametric Tolerance Tubes

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Abstract

Tolerance intervals and tolerance regions are important tools for process monitoring or statistical quality control of univariate and multivariate data, respectively. We discuss their generalization to tolerance tubes in the infinite dimensional setting for functional data. In addition to the generalizations of the commonly accepted definitions of the tolerance level of beta-content or beta-expectation, we introduce the new definition of alpha-exempt beta-expectation tolerance tube. The latter loosens the definition of beta-expectation tolerance tube by allowing alpha (pre-set using domain knowledge) portion of each functional be exempt from the requirement. Those proposed tolerance tubes are completely nonparametric and broadly applicable. We discuss their general properties, and show that the alpha exempt beta-expectation tolerance tube is particularly useful in the setting where occasional short term aberrations of the functional data are deemed acceptable (or unpreventable) and they do not cause substantive deviation of the norm. This desirable property is elaborated further and illustrated with both simulations and real applications in continuous monitoring of blood glucose level in diabetes patients as well as of aviation risk patterns of aircraft landings. This nonparametric tolerance tube can apply broadly to many domains dealing with functional data or continuous monitoring.

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